

Calculation Policy for Grace Mary Primary School

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Background

This policy has been developed by the Maths Coordinator with a specific interest in the use of Singapore methods to develop number awareness and fluency. The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

Using the concrete-pictorial-abstract approach:

Children develop an understanding of a mathematical concept through the three steps (or representation) of concrete-pictorial-abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete representation:

A pupil is first introduced to an idea or a skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation:

A pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation

The symbolic stage - a pupil is now capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$

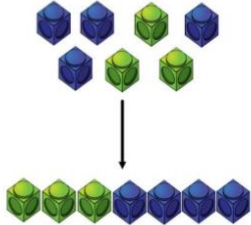
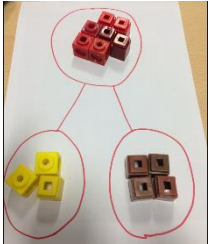
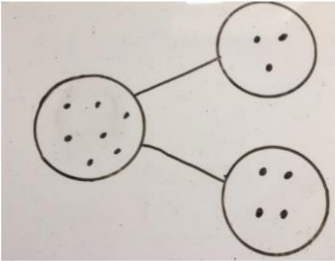
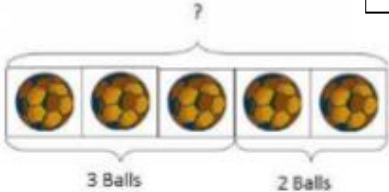
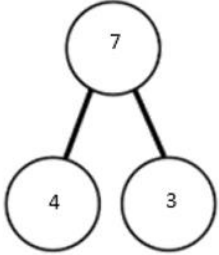
Guidance

This document provides guidance and examples for key objectives for each year group but is not to be followed as a complete planning aid as not all objectives are exemplified.

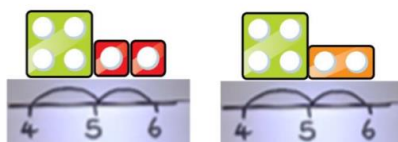
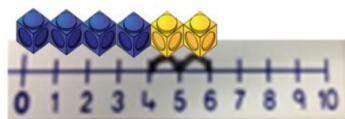
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Reception

<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Combining two parts to make a whole	Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).   Concrete representation of the part-whole model	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.   Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven. 

Starting at a bigger number and counting on

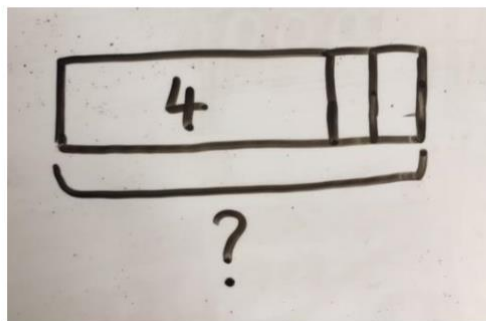


Concrete representation
for $4 + 2 =$



Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

A bar model which encourages the children to count on, rather than count all.



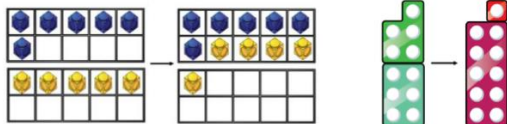
The abstract number line:
What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2?
 $4 + 2$



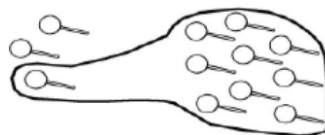
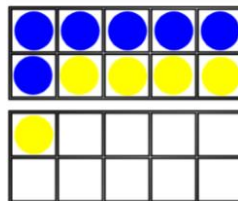
Regrouping to make 10

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Children to draw the ten frame and counters/cubes.



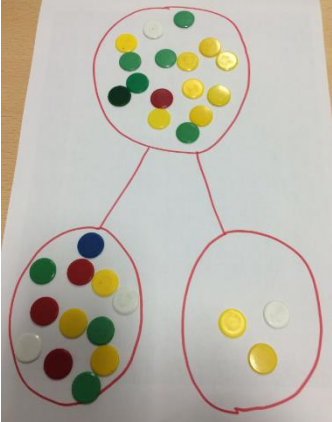
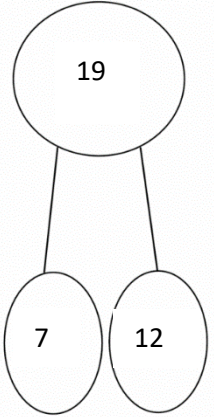
$$3 + 9 =$$

Use pictures or a number line.
Regroup or partition the smaller number to make 10.

$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

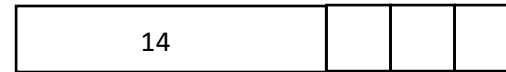
Year 1- Consolidate CPA approach from Reception but extend to addition facts to 20

<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
Combining two parts to make a whole	Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).   5 + 12 = Children combine and then count  Part whole model for 12 + 3 = 15	Bar model for 12 + 4 = 	

Starting at a bigger number and counting on



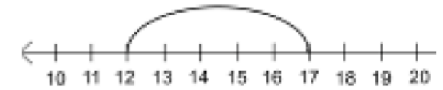
$14 + 3 = 17$
Children start with 14 in the egg trays and then add 3 more.



$$14 + 3 =$$

A bar model which encourages the children to count on, not all.

$$12 + 5 = 17$$

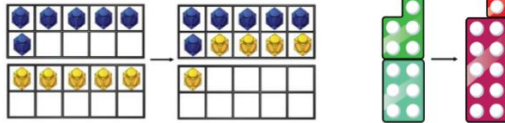


Start at the larger number on the number line and count on in ones or in one jump to find the answer.

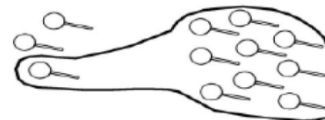
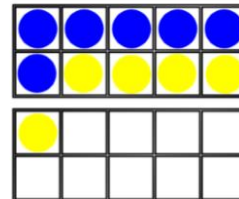
Regrouping to make 10

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Children to draw the ten frame and counters/cubes.



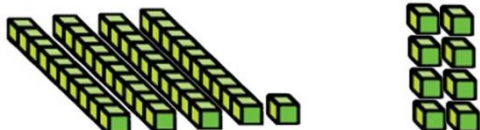
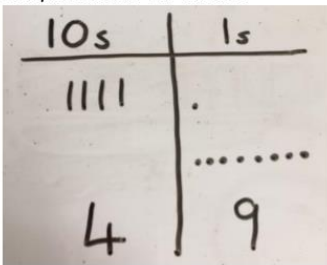
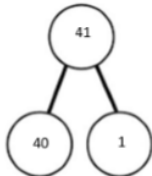
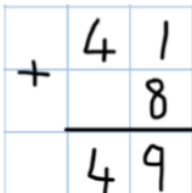
$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number to make 10.

$$7 + 4 = 11$$

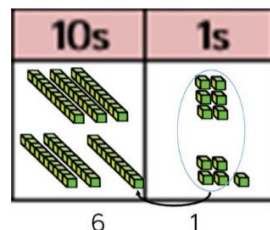
If I am at seven, how many more do I need to make 10. How many more do I add on now?

Year 2

Objective and strategies	Concrete	Pictorial	Abstract						
TO + O	TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8  Use base 10 blocks before moving onto place value counters 24 + 3 = <table><tr><th>T</th><th>O</th></tr><tr><td>  </td><td>  </td></tr><tr><td></td><td>  </td></tr></table>	T	O					Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
T	O								
									
									

TO + TO

TO + TO using base 10. Continue to develop understanding of partitioning and place value.
36 + 25

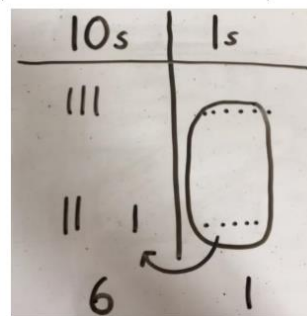


$$24 + 43 =$$

T	O

When regrouping, children need to be taught that ten ones is the same as one ten.

Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

$$36 + 25 =$$

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

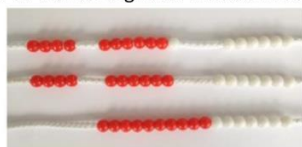
Formal method:

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ 1 \end{array}$$

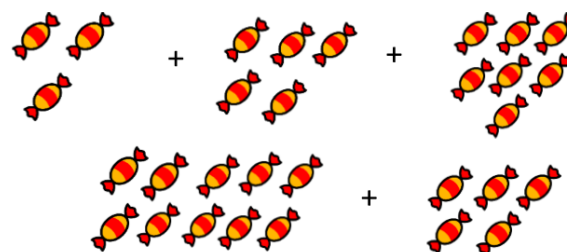
Adding three single digits

$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

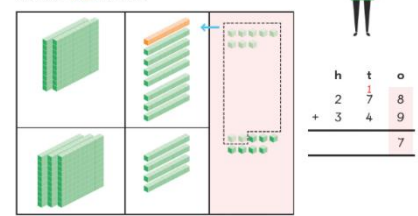
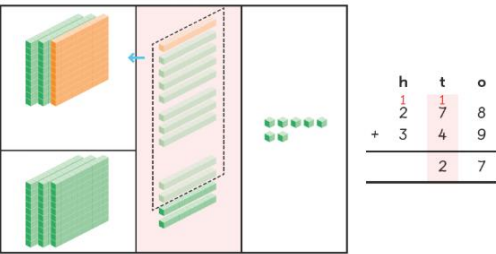
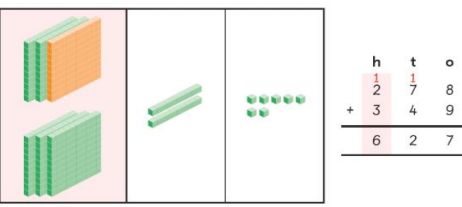
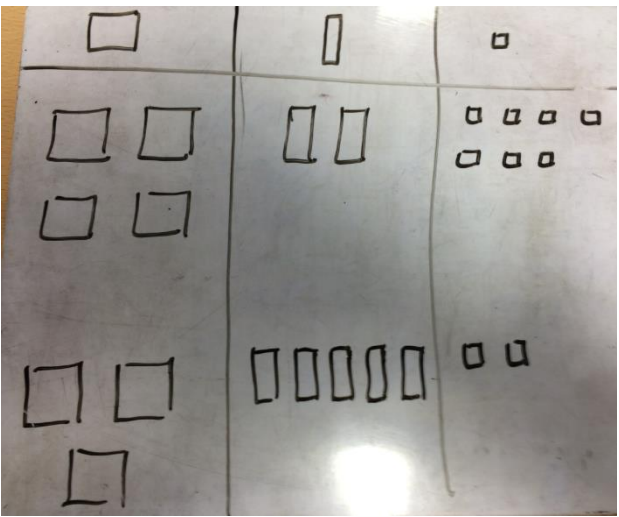


Add together three groups of objects. Draw a picture to recombine the groups to make 10.

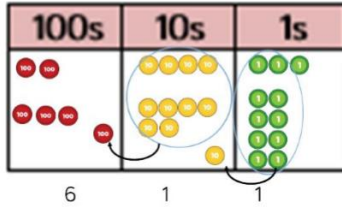
$$\begin{array}{l} 4 + 7 + 6 = 10 + 7 \\ 10 \\ = 17 \end{array}$$

Combine the two numbers that make 10 and then add on the remainder.

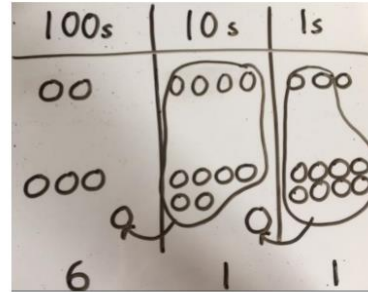
Year 3

Objective and strategies	Concrete	Pictorial	Abstract																																																
Add two three digit numbers	Add 278 and 349. Step 1 Add the ones. 8 ones + 9 ones = 17 ones Regroup the ones. 17 ones = 1 ten + 7 ones The sum is the total of 278 and 349.  <table border="1" data-bbox="672 534 784 630"> <thead> <tr> <th></th> <th>h</th> <th>t</th> <th>o</th> </tr> </thead> <tbody> <tr> <td></td> <td>2</td> <td>7</td> <td>8</td> </tr> <tr> <td>+</td> <td>3</td> <td>4</td> <td>9</td> </tr> <tr> <td></td> <td></td> <td></td> <td>7</td> </tr> </tbody> </table> Step 2 Add the tens. 1 ten + 7 tens + 4 tens = 12 tens Regroup the tens. 12 tens = 1 hundred + 2 tens  <table border="1" data-bbox="728 853 862 973"> <thead> <tr> <th></th> <th>h</th> <th>t</th> <th>o</th> </tr> </thead> <tbody> <tr> <td></td> <td>1</td> <td>7</td> <td>8</td> </tr> <tr> <td>+</td> <td>3</td> <td>4</td> <td>9</td> </tr> <tr> <td></td> <td>2</td> <td></td> <td>7</td> </tr> </tbody> </table> Step 3 Add the hundreds. 1 hundred + 2 hundreds + 3 hundreds = 6 hundreds  <table border="1" data-bbox="705 1157 828 1268"> <thead> <tr> <th></th> <th>h</th> <th>t</th> <th>o</th> </tr> </thead> <tbody> <tr> <td></td> <td>1</td> <td>7</td> <td>8</td> </tr> <tr> <td>+</td> <td>3</td> <td>4</td> <td>9</td> </tr> <tr> <td></td> <td>6</td> <td>2</td> <td>7</td> </tr> </tbody> </table> 278 + 349 = 627		h	t	o		2	7	8	+	3	4	9				7		h	t	o		1	7	8	+	3	4	9		2		7		h	t	o		1	7	8	+	3	4	9		6	2	7	Children draw their own representation of the addition using hundred, tens and ones blocks. 	243 +368 611 1 1
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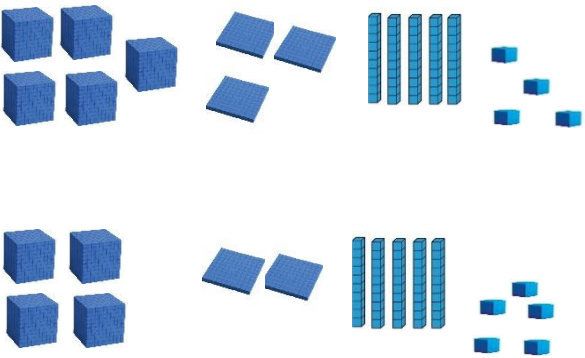
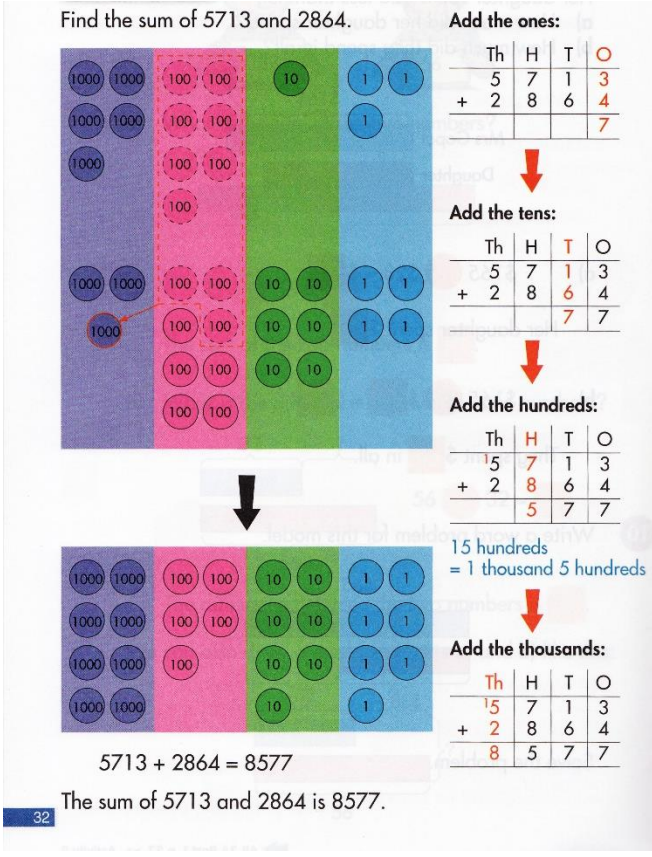
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



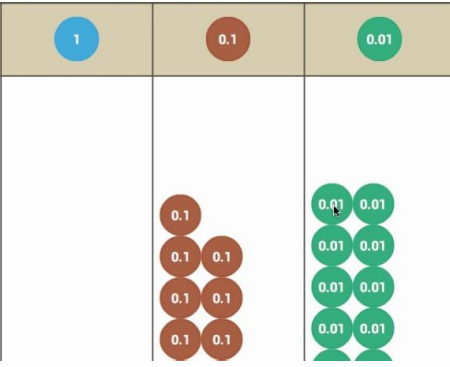
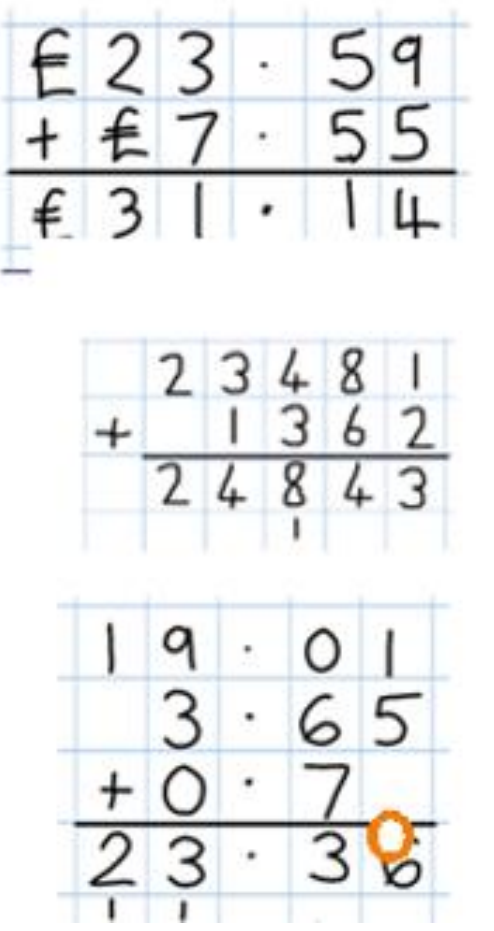
Children to represent the counters in a place value chart, circling when they make an exchange.



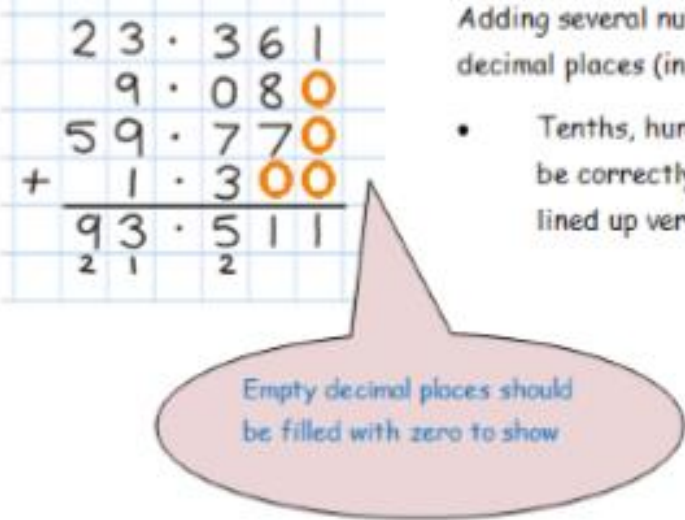
Year 4

Objective and strategies	Concrete	Pictorial	Abstract
Add two four digit numbers	 <u>Use of base 10 apparatus to add. Children need to be taught to exchange e.g. ten 10s = one 100.</u>	Find the sum of 5713 and 2864.  32 The sum of 5713 and 2864 is 8577.	4365 + 5146 = $ \begin{array}{r} 4365 \\ + 5146 \\ \hline 9511 \end{array} $

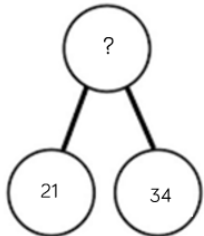
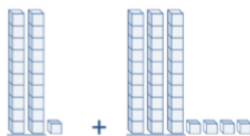
Year 5

Objective and strategies	Concrete	Pictorial	Abstract
<u>Adding numbers with more than four digits including decimals.</u>		<u>Use of place value discs to support with understanding</u>  <u>The above diagram shows the number 0.78.</u>	

Year 6

Objective and strategies	Concrete	Pictorial	Abstract
Adding several numbers with up to three decimal places		Adding several numbers with different numbers of decimal places (including money and measures): Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.	

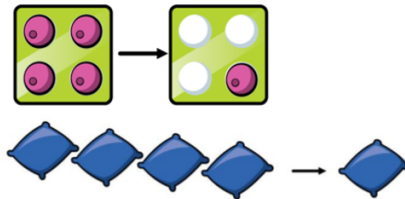
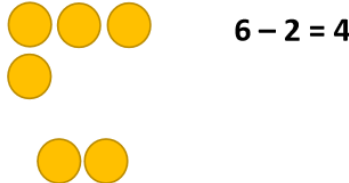
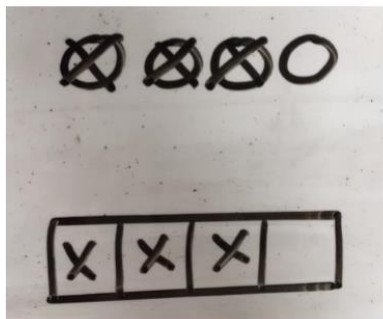
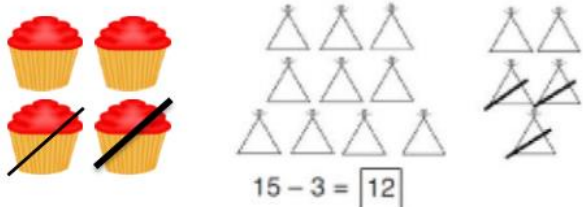
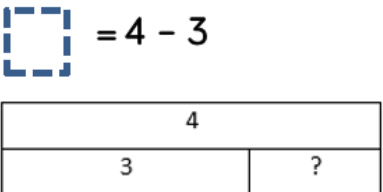
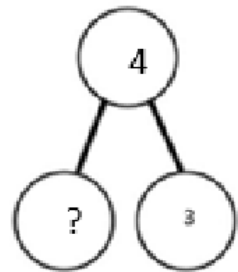
Conceptual variation; different ways to ask children to solve $21 + 34$

 <table data-bbox="262 1383 618 1458"><tr><td colspan="2">?</td></tr><tr><td>21</td><td>34</td></tr></table>	?		21	34	Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total? $21 + 34 = 55$. Prove it	$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$ $21 + 34 =$ $= 21 + 34$ Calculate the sum of twenty-one and thirty-four.	 Missing digit problems: <table data-bbox="1568 1364 1812 1549"><tr><th>10s</th><th>1s</th></tr><tr><td></td><td></td></tr><tr><td></td><td>?</td></tr><tr><td>?</td><td>5</td></tr></table>	10s	1s				?	?	5
?															
21	34														
10s	1s														
	?														
?	5														

Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

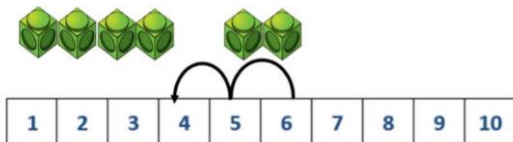
Reception

Objective and strategies	Concrete	Pictorial	Abstract
Taking away ones	Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). $4 - 3 = 1$  Use physical objects, counters, cubes etc to show how objects can be taken away. 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.  Cross out drawn objects to show what has been taken away. 	$4 - 3 =$  

Counting back

Counting back (using number lines or number tracks) children start with 6 and count back 2.

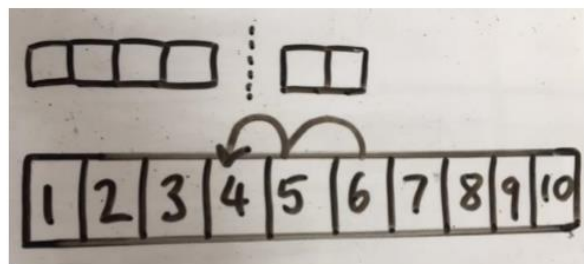
$$6 - 2 = 4$$



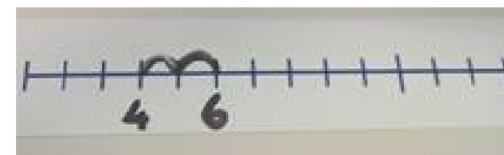
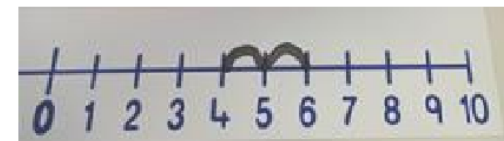
Use counters and move them away from the group as you take them away counting backwards as you go.



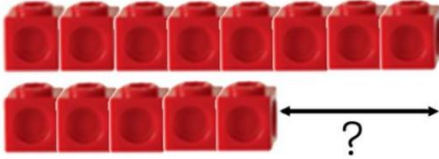
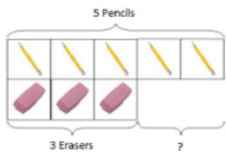
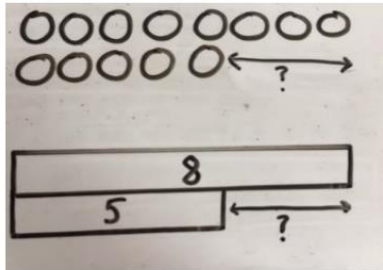
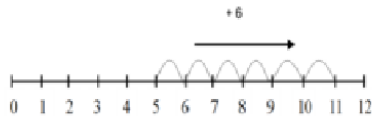
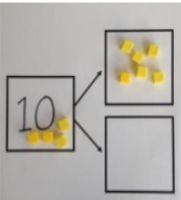
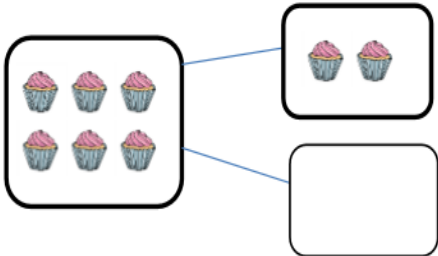
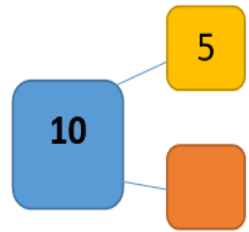
Children to represent what they see pictorially e.g.



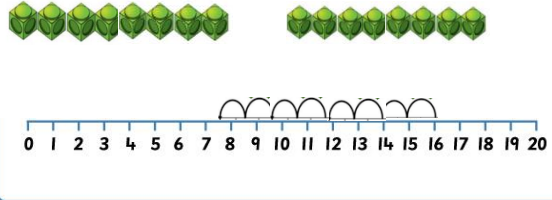
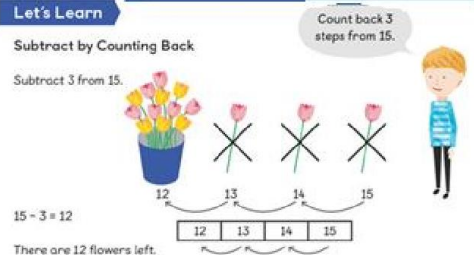
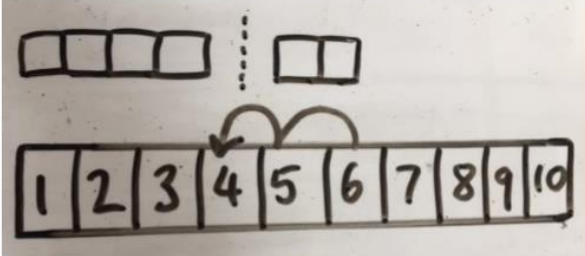
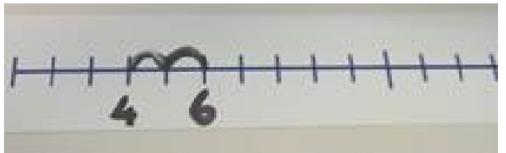
Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line



Put 8 in your head, count back 4. What number are you at?.

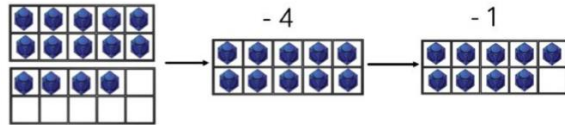
Finding the difference	Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5.   Use basic bar models with items to find the difference.	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 - 5, the difference is Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference.  Count on to find the difference.
Part Part Whole	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

Year 1- Consolidate CPA approach from Reception but extend to subtraction facts

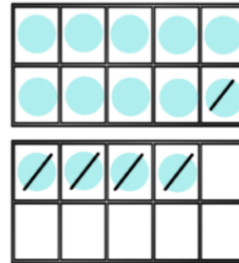
Objective and strategies	Concrete	Pictorial	Abstract
Counting back	Counting back- (using number lines or number tracks children start with 16 and count back 8 $16 - 8 = 8$)  The image shows two rows of green cubes. The first row has 16 cubes, and the second row has 8 cubes. Below them is a number line from 0 to 20. A bracket is drawn from 16 to 8, with four curved arrows indicating a count back of 8 units. Let's Learn Subtract by Counting Back Subtract 3 from 15.  The image shows a boy counting back 3 steps from 15. There are 15 flowers in a pot, and 3 are crossed out. A number line shows 12, 13, 14, and 15. A bracket is drawn from 15 to 12, with three curved arrows indicating a count back of 3 units. The text says: $15 - 3 = 12$ and 'There are 12 flowers left.'	Children to represent what they see pictorially e.g.  The image shows a number track from 1 to 10. A bracket is drawn from 10 to 7, with three curved arrows indicating a count back of 3 units. Above the track, there are two boxes, one with 3 and one with 2, separated by a dashed line. Numbers to 20	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line   The first image shows a number line from 0 to 10. A bracket is drawn from 10 to 7, with three curved arrows indicating a count back of 3 units. The second image shows a number line from 0 to 10. A bracket is drawn from 6 to 2, with four curved arrows indicating a count back of 4 units. Put 17 in your head. Count back 4. What number are you at?

Make 10

Making 10 using ten frames.
 $14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



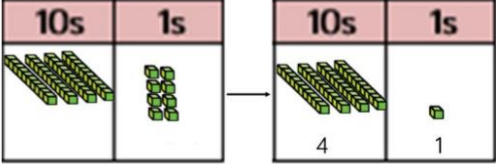
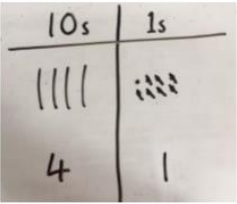
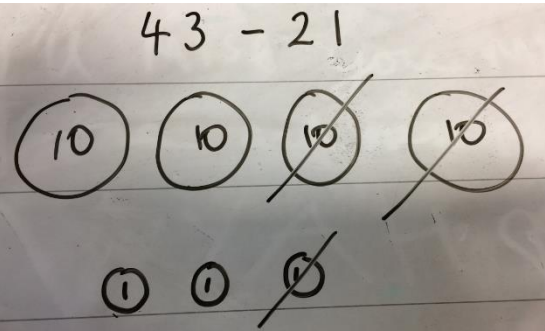
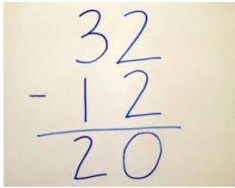
Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

A tree diagram showing the partitioning of the subtrahend 5 into 4 and 1. The number 5 is at the top, with two lines branching down to the numbers 4 and 1.

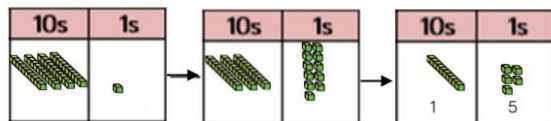
$$14 - 4 = 10$$
$$10 - 1 = 9$$

Year 2

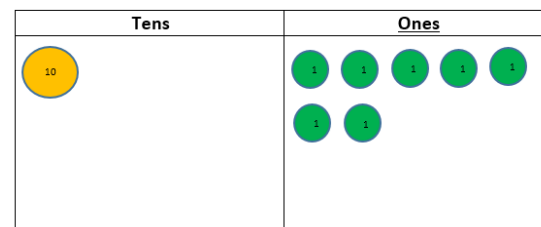
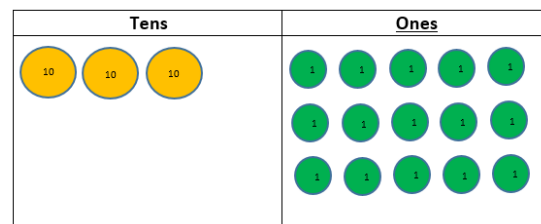
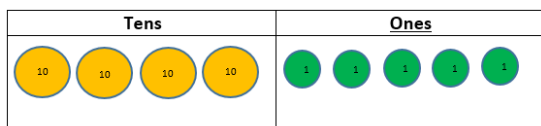
<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
<u>Column method- no exchanging</u>	Column method using base 10. 48-7  Use place value discs to subtract 	Children to represent the base 10 pictorially.  <u>Children to represent place value discs pictorially</u> 	$47 - 24 = 23$ $\begin{array}{r} 47 \\ - 24 \\ \hline 20 + 7 \\ \hline 20 + 3 \\ \hline \end{array}$ This will lead to a clear written column subtraction. 

**Column method
with
exchanging**

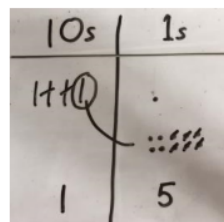
Column method using base 10 and having to exchange.
41 – 26



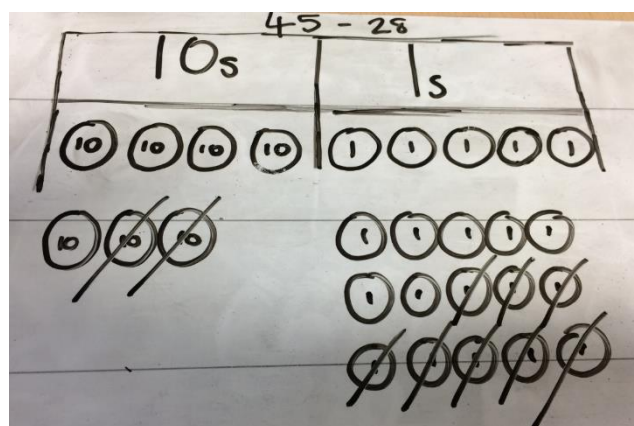
Column method using place value counters 45 – 28



Represent the base 10 pictorially, remembering to show the exchange.



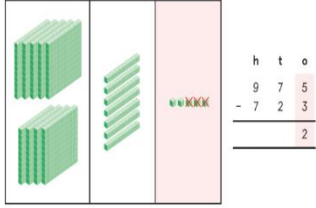
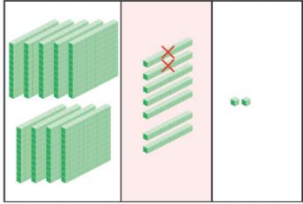
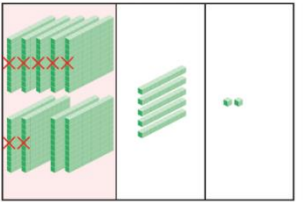
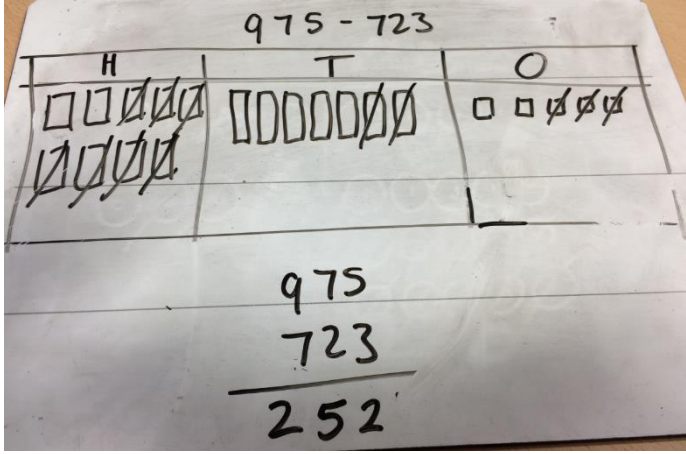
Represent the place value disks pictorially.



Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.

	3	4	1
-		2	6
		1	5

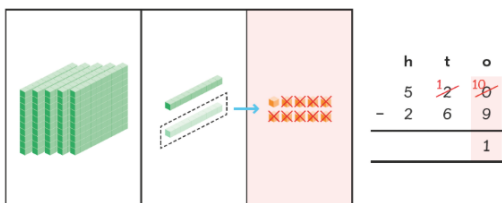
Year 3

Objective and strategies	Concrete	Pictorial	Abstract																
Column method no exchanging	Subtract 723 from 975. Step 1 Subtract the ones. 5 ones - 3 ones = 2 ones  Step 2 Subtract the tens. 7 tens - 2 tens = 5 tens  Step 3 Subtract the hundreds. 9 hundreds - 7 hundreds = 2 hundreds  975 - 723 = 252 Children then progress onto using place value counters using strategy outlined in year 2 guidance.	Children represent base ten pictorially  Children then progress onto using place value counters using strategy outlined in year 2 guidance.	<table> <tr> <th></th><th>h</th><th>t</th><th>o</th></tr> <tr> <td></td><td>9</td><td>7</td><td>5</td></tr> <tr> <td>-</td><td>7</td><td>2</td><td>3</td></tr> <tr> <td></td><td>2</td><td>5</td><td>2</td></tr> </table>		h	t	o		9	7	5	-	7	2	3		2	5	2
	h	t	o																
	9	7	5																
-	7	2	3																
	2	5	2																

**Column method
with
exchanging**

520 – 269 =

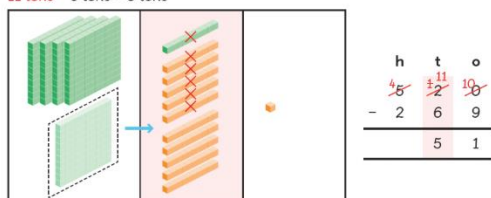
Step 1



Step 2 Regroup 1 hundred into 10 tens.

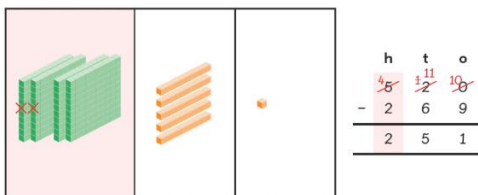
Subtract the tens.

11 tens – 6 tens = 5 tens



Step 3 Subtract the hundreds.

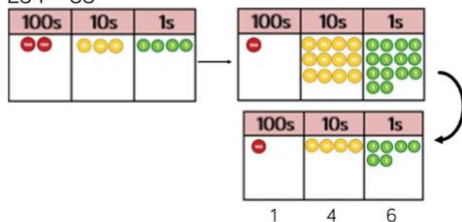
4 hundreds – 2 hundreds = 2 hundreds



520 – 269 = 251

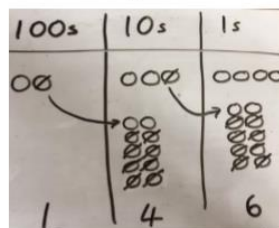
Column method using place value counters.

234 – 88



**Children represent base ten apparatus pictorially as
outlined above**

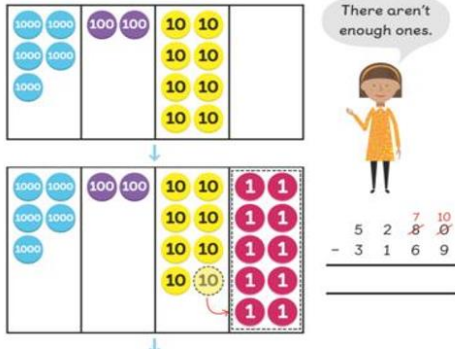
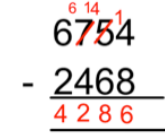
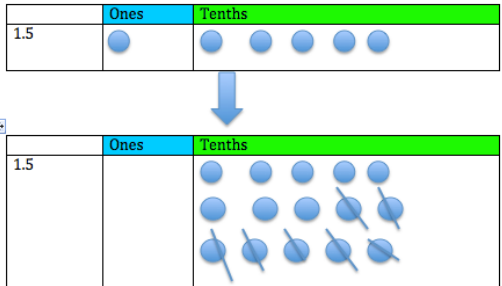
Represent the place value counters pictorially;
remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

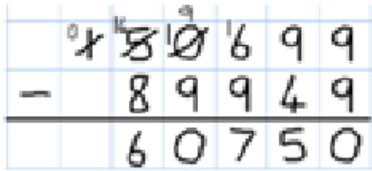
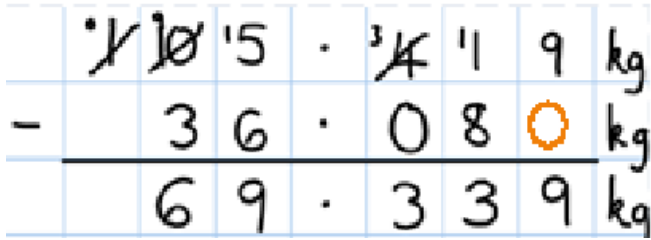
Year 4

Objective and strategies	Concrete	Pictorial	Abstract
Column method not exchanging and exchanging	Base 10 apparatus to subtract 5436 - 3412  Take away the physical apparatus. Progress onto using place value disks: 5280 - 3169 	Pictorial representation of base 10 apparatus and place value disks as outlined in the year 3 guidance above.	6754 - 2468 =  1.5m - 0.7m =  So 1.5m - 0.7m = 0.8m

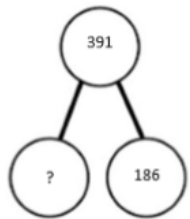
Year 5

Objective and strategies	Concrete	Pictorial	Abstract
<u>Subtract with at least four digit numbers and decimals</u> <u>(children may do this concrete or pictorially first if conceptual understanding is not there)</u>		Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point. $\begin{array}{r} \overset{2}{\cancel{3}} \overset{10}{\cancel{0}} \overset{1}{\cancel{0}} \overset{4}{\cancel{5}} \overset{1}{\cancel{6}} \\ - 2128 \\ \hline 28928 \end{array}$  $\begin{array}{r} \overset{6}{\cancel{7}} \overset{10}{\cancel{6}} \overset{8}{\cancel{9}} \cdot \overset{1}{\cancel{0}} \\ - 372 \cdot 5 \\ \hline 6796 \cdot 5 \end{array}$	

Year 6

Objective and strategies	Concrete	Pictorial	Abstract
<u>Subtracting increasingly large and complex numbers and decimal values (children may do this concrete or pictorially first if conceptual understanding is not there)</u>	 		Very important to use in a range of contexts- measures and money.

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\boxed{} = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

What is 186 less than 391?

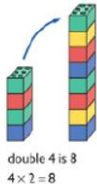
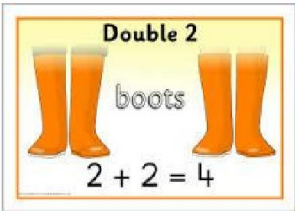
Missing digit calculations

$$\begin{array}{r} 39\boxed{} \\ -\boxed{}\boxed{}6 \\ \hline \boxed{}05 \end{array}$$

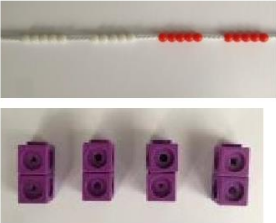
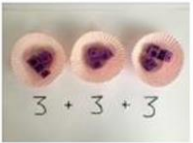
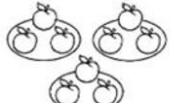
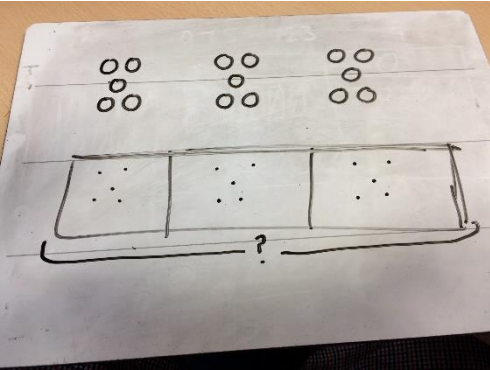
Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Reception

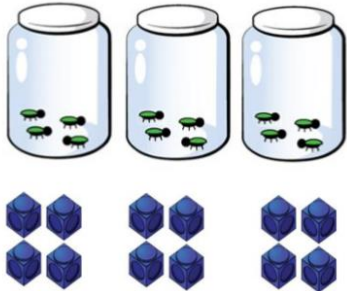
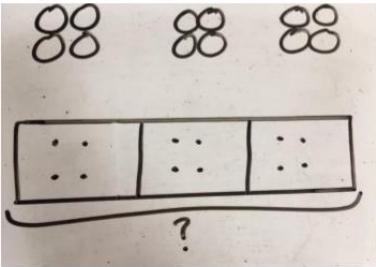
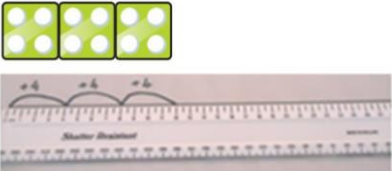
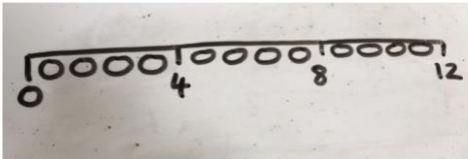
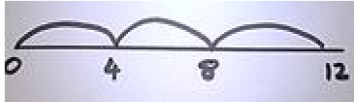
<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
<u>Doubling</u>	Use practical activities to show how to double a number.  double 4 is 8 $4 \times 2 = 8$  Double 2 boots $2 + 2 = 4$	Draw pictures to show how to double a number. Double 4 is 8 	<u>Children recite doubles to 20 by memory</u>

Year 1

Objective and strategies	Concrete	Pictorial	Abstract
<u>Counting in multiples of 2, 5 and 10</u>	 Count in multiples supported by concrete objects in equal groups.	Use pictures to continue support in counting in multiples 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
<u>Repeated addition</u>	 Use different objects to add equal groups.  $3 + 3 + 3$  How many apples are there altogether?	Children to represent the practical resources in a picture and use a bar model 	$3 \times 5 = 15$ $5 + 5 + 5 = 15$

<u>Arrays</u>	Create arrays using counters/ cubes to show multiplication sentences. 	 $4 \times 2 = 8$ $2 \times 4 = 8$  $2 \times 4 = 8$ $4 \times 2 = 8$ Draw arrays in different rotations to find commutative multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition.
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Year 2

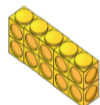
Objective and strategies	Concrete	Pictorial	Abstract
<u>Repeated grouping/repeated addition</u>	3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$
<u>Number lines to show repeated groups</u>	Number lines to show repeated groups- 3×4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. $3 \times 4 = 12$ 

Use arrays to
illustrate
commutativity

Use arrays to illustrate commutativity counters and other objects can also be used.
 $2 \times 5 = 5 \times 2$

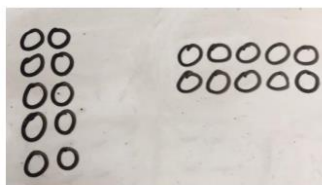


2 lots of 5



5 lots of 2

Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

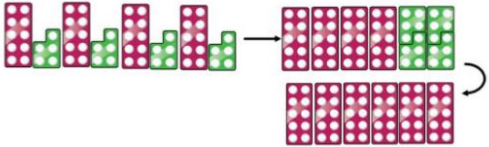
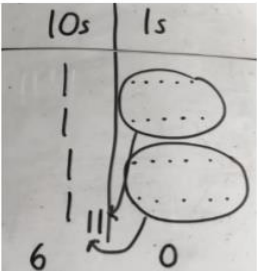
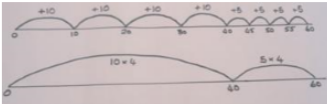
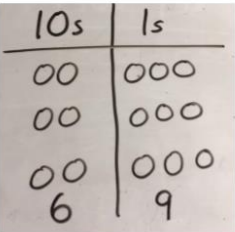
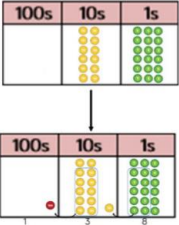
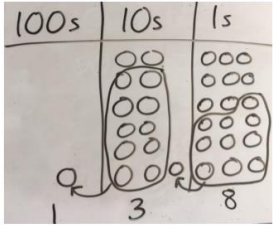
$$10 = 2 \times 5$$

$$5 \times 2 = 10$$

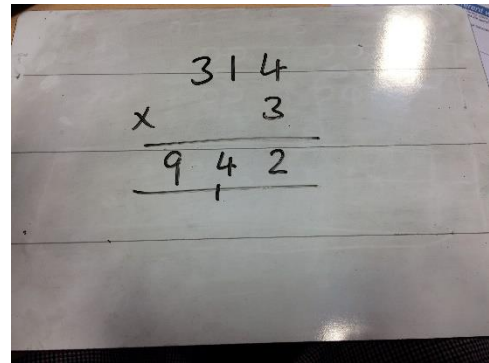
$$2 + 2 + 2 + 2 + 2 = 10$$

$$10 = 5 + 5$$

Year 3

Objective and strategies	Concrete	Pictorial	Abstract
Partition to multiply	Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. $\begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$ $\begin{array}{l} 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$ A number line can also be used 
Formal column method	Formal column method with place value counters (base 10 can also be used.) 3×23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. $\begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array}$ $\begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$ $\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$
Formal column method	Formal column method with place value counters. 6×23 	Children to represent the counters/base 10, pictorially e.g. the image below. 	Formal written method $6 \times 23 =$ $\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 1 \quad 1 \end{array}$

Year 4

<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
<u>4 digit x 1 digit</u>	Repeat strategies listed above in year 3 objectives if required. Most children should be quickly moving on to the abstract stage of learning.	Repeat strategies listed above in year 3 objectives if required. Most children should be quickly moving on to the abstract stage of learning.	$ \begin{array}{r} 314 \\ \times 3 \\ \hline 12 \quad (3 \times 4) \\ 30 \quad (3 \times 10) \\ + 900 \quad (3 \times 300) \\ \hline 942 \end{array} $ Progressing onto...  The image shows a close-up of a child's handwriting on lined paper. It displays the multiplication of 314 by 3. The numbers are written as follows: 314 on the top line, a multiplication sign (x) to the left, and 3 on the second line. A horizontal line is drawn below the 3. The product 942 is written on the third line, with a small vertical tick mark under the 2. The handwriting is clear and legible.

Year 5 and 6

Objective and strategies	Concrete	Pictorial	Abstract
Column method multiplying by a 2 digit number		Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. $\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$ $\begin{array}{r} 74 \\ \times 63 \\ \hline 12 \\ 210 \\ 240 \\ + 4200 \\ \hline 4662 \end{array}$ This moves to the more compact method. $\begin{array}{r} 231 \\ 1342 \\ \times 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \\ 1 \end{array}$	

Conceptual variation; different ways to ask children to solve 6×23



?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$6 \times 23 =$

$\square = 6 \times 23$

$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array}$

$\begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$

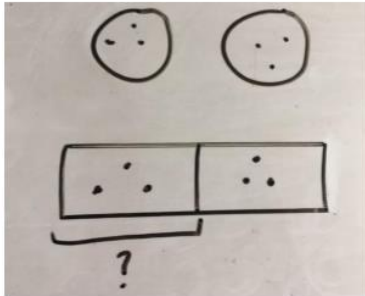
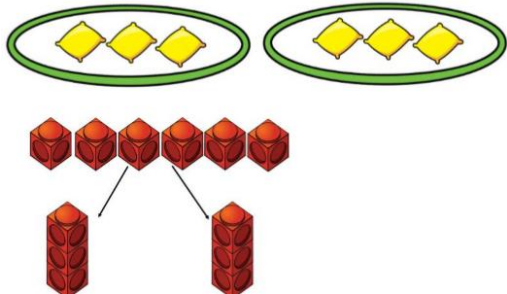
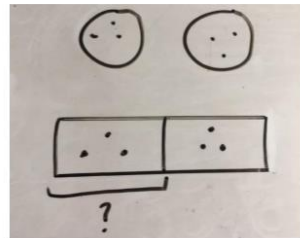
What is the calculation?
What is the product?

100s	10s	1s
		

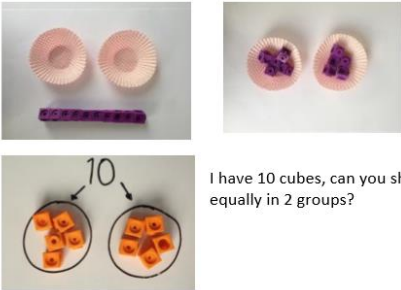
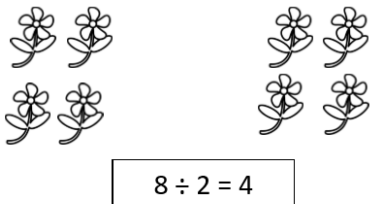
Calculation policy: Division

Key language: share, group, divide, divided by, half.

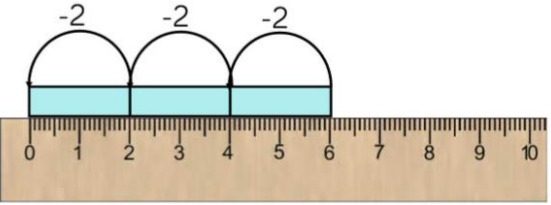
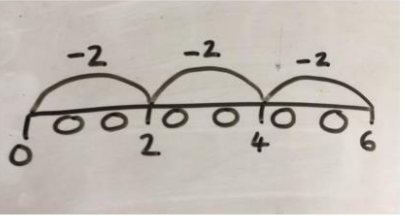
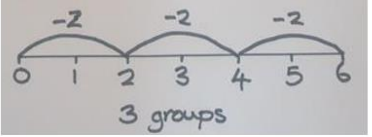
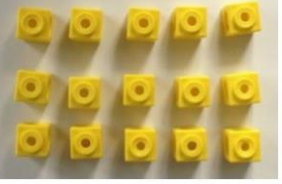
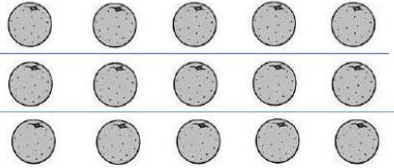
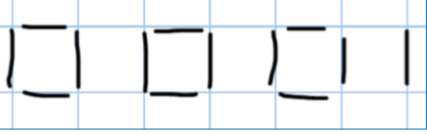
Rec

Objective and strategies	Concrete	Pictorial	Abstract		
Halving	Use of concrete resources to support conceptual understanding. 	Represent the sharing pictorially. 	Children to recite halves of numbers up to 20 from memory (after lots of practical work first).		
Sharing using a range of objects		Represent the sharing pictorially. 	Share objects into equal groups. Use related vocabulary. Activities might include: - Sharing of milk at break time. - Sharing sweets on a child's birthday. - Sharing activities in the home corner. $6 \div 2 = 3$ <table data-bbox="1619 1208 1948 1257"><tr><td>3</td><td>3</td></tr></table> Children should also be encouraged to their 2 times tables facts.	3	3
3	3				

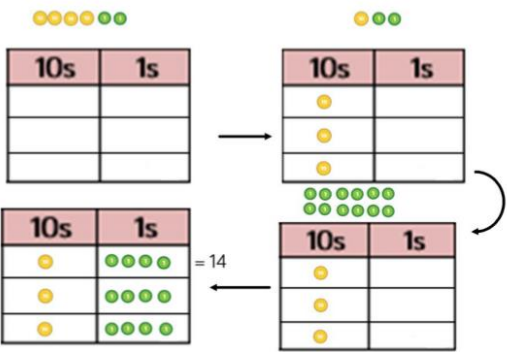
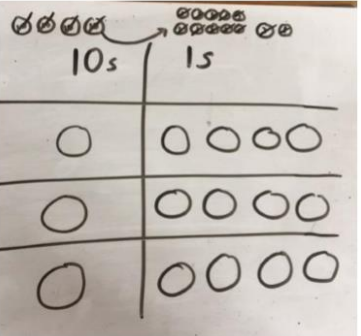
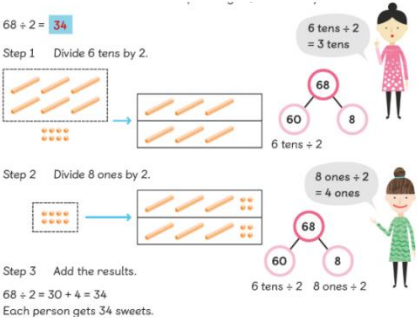
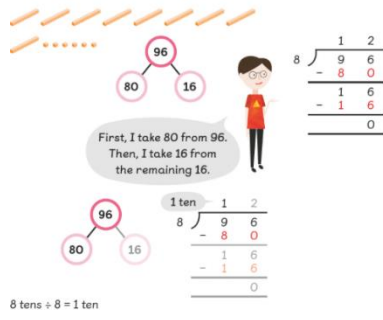
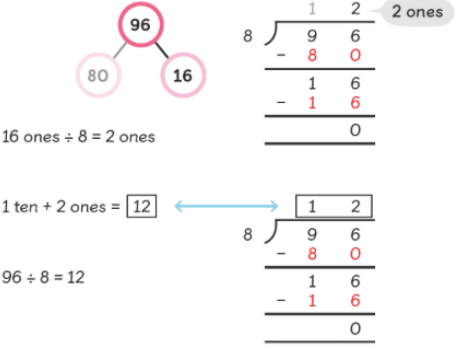
Year 1

<u>Objective and strategies</u>	<u>Concrete</u>	<u>Pictorial</u>	<u>Abstract</u>
<u>Sharing objects into groups</u>	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$

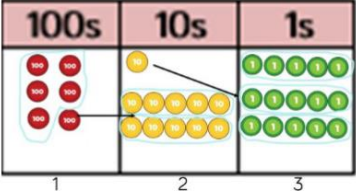
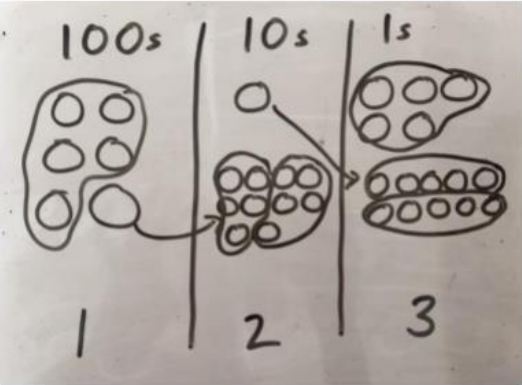
Year 2

Objective and strategies	Concrete	Pictorial	Abstract
<u>Repeated subtraction</u>	Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 
<u>Division with arrays</u>	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. $4 \times 5 = 20$ $5 \times 4 = 20$ $20/4 = 5$ $20/5 = 4$
<u>2d/1d with remainders</u>	$2d \div 1d$ with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. $13 \div 4$ Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	$13 \div 4 = 3$ remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 

Year 3

Objective and strategies	Concrete	Pictorial	Abstract
Sharing using place value counters	Sharing using place value counters. $42 \div 3 = 14$ 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$
Short division	 $68 \div 2 = 34$ Step 1 Divide 6 tens by 2. $6 \text{ tens} \div 2 = 3 \text{ tens}$ $6 \text{ tens} \div 2 = 30$ Step 2 Divide 8 ones by 2. $8 \text{ ones} \div 2 = 4 \text{ ones}$ $8 \text{ ones} \div 2 = 4$ Step 3 Add the results. $68 \div 2 = 30 + 4 = 34$ Each person gets 34 sweets.	 $96 \div 8$ First, I take 80 from 96. Then, I take 16 from the remaining 16. $8 \text{ tens} \div 8 = 1 \text{ ten}$	 $16 \text{ ones} \div 8 = 2 \text{ ones}$ $1 \text{ ten} + 2 \text{ ones} = 12$ $96 \div 8 = 12$

Year 4

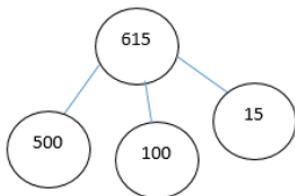
Objective and strategies	Concrete	Pictorial	Abstract
Short division	Short division using place value counters to group. $615 \div 5$  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially. 	Children to the calculation using the short division scaffold. $ \begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array} $

Year 5 and 6

Objective and strategies	Concrete	Pictorial	Abstract																															
Long division	Long division using place value counters $2544 \div 12$ <table><tr><td>1000s</td><td>100s</td><td>10s</td><td>1s</td></tr><tr><td></td><td></td><td></td><td></td></tr></table> We can't group 2 thousands into groups of 12 so will exchange them. <table><tr><td>1000s</td><td>100s</td><td>10s</td><td>1s</td></tr><tr><td></td><td></td><td></td><td></td></tr></table> We can group 24 hundreds into groups of 12 which leaves with 1 hundred. $\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$ <table><tr><td>1000s</td><td>100s</td><td>10s</td><td>1s</td></tr><tr><td></td><td></td><td></td><td></td></tr></table> After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens. $\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$ <table><tr><td>1000s</td><td>100s</td><td>10s</td><td>1s</td></tr><tr><td></td><td></td><td></td><td></td></tr></table> After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder. $\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$	1000s	100s	10s	1s					1000s	100s	10s	1s					1000s	100s	10s	1s					1000s	100s	10s	1s					Divide numbers up to 4 digits by a one-digit or two digit number using the formal written method of short division and interpret remainders appropriately for the context $432 \div 15$ becomes $\begin{array}{r} 28 \text{ r } 12 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$ $432 \div 15$ becomes $\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$ $\frac{12}{15} = \frac{4}{5}$ Answer: $28 \frac{4}{5}$ $432 \div 15$ becomes $\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \downarrow \\ 132 \\ \underline{120} \downarrow \\ 120 \\ \underline{120} \downarrow \\ 0 \end{array}$ Answer: 28.8
1000s	100s	10s	1s																															
1000s	100s	10s	1s																															
1000s	100s	10s	1s																															
1000s	100s	10s	1s																															

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\boxed{} = 615 \div 5$$

What is the calculation?
What is the answer?

